10/2/25

Lecture 11

Example

$$R = \{(x,y) : x = y\} \subseteq \mathbb{R} \times \mathbb{R}$$

$$S = \{(x,y) : x \leq y\} \subseteq \mathbb{R} \times \mathbb{R}$$

$$\forall x \forall z (x,z) \in R \circ S \equiv \exists y \, (xSy \wedge yRz)$$

$$\Downarrow$$

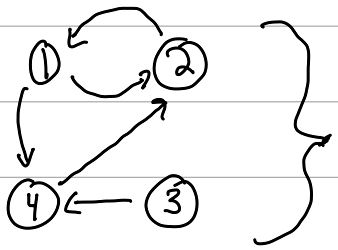
$$\equiv \exists y \, (x \leq y \wedge y = z)$$

$$\equiv \exists y \, (x \leq z)$$

The composition means $\forall x \forall z \, (x \leq z)$

Also this composition is equivalent to the relation S.

Graph Powers



$G = (V, E)$

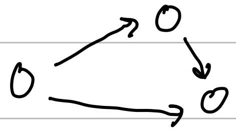
- V is set of vertices
- E is set of edges

$$E = \{(1,2), (2,1), (1,4), (4,2), (3,4)\}$$

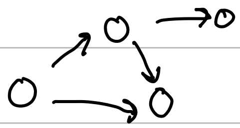
$$E^2 = E \circ E = \{(1,1), (2,2), (2,4), (1,2), (4,1), (3,2)\}$$

All walks in E^2 are of length 2.

New Example



Transative!

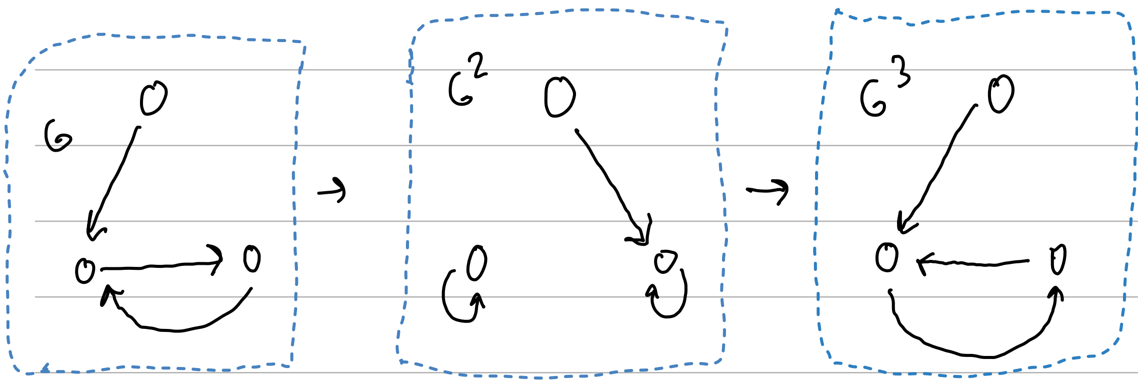


Not transative!

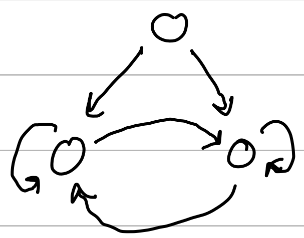
New example

① ② $G \cup G^2 \cup G^3 \cup G^4$

④ ③ This would include all the walk of length 1,2,3 or 4 in G.

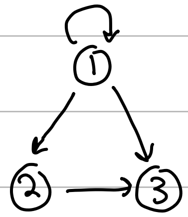


$G \cup G^2 \cup G^3 = G^+$



6.6 Matrix multiplication and graph powers

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$



$A_{13} = A[1][3]$

Boolean Addition	Boolean Multiplication
$1+1=1$	$1 \cdot 1 = 1$
$1+0=1$	$1 \cdot 0 = 0$
$0+1=1$	$0 \cdot 1 = 0$
$0+0=0$	$0 \cdot 0 = 0$

Boolean Matrix Multiplication

$$A = \begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 0\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

☆ • each one in A^2 tells you that a walk exists of length 2 that start and end on the corresponding nodes to the entry substrics.

☆ • If you did A^2 with normal matrix multiplication you would get $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

- The 2 in A^2 at entry (1,3) tells you there are <u>2</u> <u>distinct</u> walks that are length 2 from node 1 to node 3.